Spheres & Inequalities

Friday, May 5, 2023 8:56 AM

sphere of radius R & center (c_1, c_2, c_3) :

$$(x-c_1)^2 + (y-c_2)^2 + (z+c_3)^2 = R^2$$

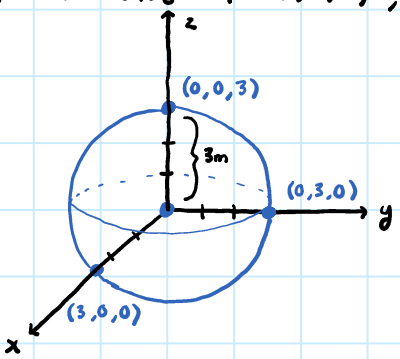
* all points (x,y,z) @ distance R from center *

ex 1) draw $S = \{x^2 + y^2 + z^2 = 9\}$ & decide if $(1,0,0)$, $(2,1,2)$ & $(0,0,327)$ belong

- $(1,0,0)$: $1^2 + 0^2 + 0^2 = 9$ ✗ ($<$ → inside)
- $(2,1,2)$: $2^2 + 1^2 + 2^2 = 9$ ✓ (on sphere)
- $(0,0,327)$: $0^2 + 0^2 + 327^2 = 9$ ✗ ($>$ → outside)

center : $(0,0,0)$ ← origin

$r = 3$



inequalities : geometrically an equation with "=" describes surface seperating space into 2 pieces

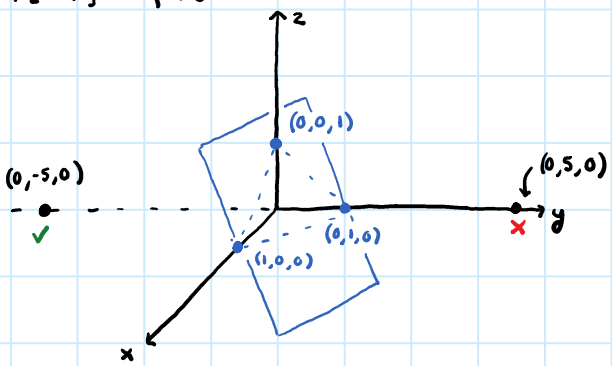
if given $\leq$, $<$, $>$, $\geq$ ...

1st) analyze equality case → get surface & 2 pieces of space

and) sample points in inequality to decide which (of 2) piece

ex 2) describe (& draw) $\{x + y + z < 1\}$

1st) note $\{x + y + z = 1\}$ = plane



$0 - 5 + 0 < 1$ ✓

so we get back half

$0 + 5 + 0 < 1$ ✗

intersections : more than 1 equation $(=, \leq, <, >, \geq)$

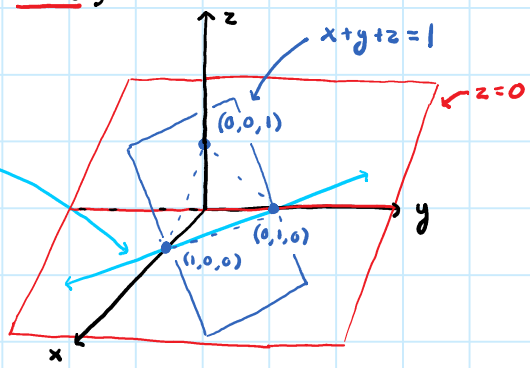
ex 3) describe & draw points in space such that

$\{x + y + z = 1\}$ and $\{z = 0\}$

plane — written as comma — plane

ex 3) describe $\{x+y+z=1 \;,\; z=0\}$

intersection is line thru $(1,0,0)$ & $(0,1,0)$



ex 4) draw $\{x^2+y^2+z^2=9 \;,\; y=2\}$

$x^2+y^2+z^2=9$: sphere

$y=2$: plane

bc only distinct y value

$(a,b,c) = (0,1,0)$ & contains $(0,a,0)$

a,b,c — x,y,z